AD-A146 256 APPLICATION OF SUMMABILITY METHODS TO JACKSON THEOREMS FOR SPLINES(U) HOWARD UNIV WASHINGTON DC DEPT OF MATHEMATICS L A RAPHAEL 30 JUN 84 ARO-20486.4-MA 1/1

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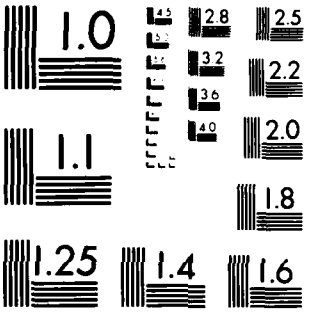

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18. SUPPLEMENTARY NOTES

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19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

1. best local approximation, several variables, L^p-norm.
2. Sturm-Liouville system; eigenfunction expansions; summability, elliptic operators, equisummability; resolvent.
3. spline, Jackson-type theorem.
4. absolute summability, Abel and Stieltjes summability methods.

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This project involved research in four mathematical areas:

1. Best local approximation in several variables for single and multi-point data sets. Hermite interpolation schemes are a special case of these schemes.
2. Analytic summability theory of eigenfunction expansions associated with singular Sturm-Liouville systems; and equisummability of eigenfunction expansions associated with elliptic operators of the same order with leading terms positive.

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3) Jackson-type theorem for best one-sided approximation by one-dimensional splines.

4) Inclusion theorems for absolute summability of divergent integrals.

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During the period of this grant, DAAG29-83-G0002, January 1, 1983 to June 30, 1984, the principal investigator Dr. Louise Raphael of the Department of Mathematics, Howard University investigated three major problems in the areas of approximation theory and eigenfunction expansions associated with Sturm-Liouville or elliptic operators on the whole space. She wrote two papers: one on a Jackson-type theorem for splines and the other on equisummability of elliptic operators. She also co-authored two articles on best local approximation in several variables; revised a manuscript on inverting generalized Fourier transforms; is completing a co-authored paper on a regularization of an ill-posed problem in several variables; and has begun a new joint paper on analytic functions of perturbed elliptic operators. In addition, two co-authored papers written under her former ARO grant DAAG29-81-G0011, were published in 1983.

This grant provided support for the following consultants who were primary forces in developing the papers written during this period.

1. Professor Charles Chui, Texas A&M University: introduced Raphael to approximation theory and splines; principal author of two joint papers on best local approximation; motivated a paper on splines.

2. Professor Mark Kon, Boston University: introduced Raphael to elliptic operators; principal author of one joint paper on eigenfunction expansions; motivated a paper on equisummability; and presently working on perturbations of elliptic operators.

Raphael's third major collaborator is:

3. Professor Harvey Diamond, West Virginia University and Texas A&M University: co-authored, three papers - two with Chui on best local approximation and the other with Kon on regularization of an ill-posed problem.

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A list of the nine articles mentioned above with brief summary, publication status and availability of reprints/preprints now follows.

I. ACCEPTED FOR PUBLICATION.

1. With C. Chui and H. Diamond: "Best Local Approximation in Several Variables", Journal of Approximation Theory. Status: Corrected galleys returned to JAT; ARO has preprint

2. With C. Chui and H. Diamond: "On Best Data Approximation", Journal of Approximation Theory and Its Applications. Status: Waiting for galleys; ARO has preprint.

Summary: These papers develop natural optimality criteria which characterize standard approximation techniques, unify several techniques under a single theory, and provide new procedures applicable in previously ambiguous cases [CSS], [WI].

Our main motivation for investigating best local approximation is the development of a unifying principle for approximating a set of data, representing the values of a function and its derivatives at a point (or finite set of points), using a function drawn from a finite dimensional approximating space. But how does one approximate data with a function? For instance, given that there is a unique interpolant of the function values, what rationale is there for choosing that interpolant for approximating the data? What if the interpolant is unique, but all the first derivatives cannot be interpolated? To give a systematic answer we imagine that we are approximating a function not only at the points in question but in small neighborhoods of the points; then L^p approximation is a natural procedure. If, as the neighborhoods shrink, the best L^p approximants have a limit, and this limit depends only on the data, then this limit, which we call the best local

approximant, is a reasonable choice for a best approximation of the data. Using our procedures, we are able to characterize Hermite interpolation schemes, weighted least ℓ^p interpolation, and mixed Hermite and ℓ^p interpolation schemes as best local approximation procedures. In some cases the best local approximants correspond to none of these schemes and conversely, no Hermite or ℓ^p procedure seemed natural to apply in the absence of best local approximation.

3. "A Jackson-Type Theorem by Averaging Splines Bounding a Class of Differentiable Functions", Journal of Approximation Theory. Status: Waiting for galleys; ARO has preprint.

Summary: This paper proves a variation of a Jackson-type theorem by Andrew, Popov and Sendov [APS] for best one-sided approximation by splines. The theorem states how well a function can be approximated by the average of upper and lower bounding splines of a given order with respect to the $L^p[0,1]$ norm of a modified modulus of continuity. Our bounds are sharper than those of [APS] by a factor of 2^{-n}, $n = 1,2,\ldots k$, n denoting the number of times the function is differentiable.

II. RESUBMITTED FOR PUBLICATION.

4. "Equisummability of Eigenfunction Expansions under Analytic Multipliers", Journal of Mathematical Analysis and Applications. Status: Revised according to referee's suggestions; waiting for referee's new report: ARO has preprint.

Summary: This paper presents abstract criteria for equisummability of expansions in eigenfunctions of certain pairs of elliptic operators on general domains of $\mathbb{R}^n$. The question of convergence of the two summability means in the L^∞ norm is reduced essentially to showing that the difference

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of the modified resolvent operators is uniformly bounded. We apply these techniques to give a simple proof of an equisummability result of Gurarie and Kon [GK] for certain class of elliptic operators whose leading terms are positive and lower order terms have coefficients which are singular on a nowhere dense set.

We apply these criteria to show:

1. the generalized Fourier transform of an $L^2(\mathbb{R}^1)$ function f associated with certain classes of Sturm-Liouville problems is analytic summable to f(x) pointwise if and only if the Fourier transform of f is analytic summable to f(x) pointwise; and

2. the convergence of solutions of the heat and perturbed heat equations to their common initial value

5. "Resolvent Means and Inverting Generalized Four Transforms", Canadian Journal of Mathematics: Status: Revised according to referee's suggestions; waiting for new report; ARO has preprint.

Summary: For singular Sturm-Liouville systems, we consider kernels of resolvent summability means for eigenfunction expansions as scaled integrals of the associated Green's functions. We use this representation to sum inverses of generalized Fourier transforms of $L^p(0,\infty)$ $(1 \leq p \leq \infty)$ functions. Specifically, we show that the generalized S-L expansion of an $L^p(0,\infty)$ function is resolvent summable to f with respect to the $L^p(0,\infty)$ norm and pointwise on the Lebesgue set of f.

III. ALMOST COMPLETED.

6. With H. Diamond and M. Kon: "On the Pointwise Regularization of Elliptic Eigenfunction Expansions".

Summary: The direct summation of an eigenfunction expansion in the presence of perturbations on the coefficients is known to be ill-posed, in the sense that small perturbations can produce large pointwise errors in the computation of the sum. This paper is an extension of regularization or stable summation methods for eigenfunction expansions defined on general subspaces on $\mathbb{R}^n$ for the Laplacian to elliptic operators on $\mathbb{R}^n$, including ones with continuous spectra.

Our problem is for f in $L^p(\mathbb{R}^n)$ $(1 \le p \le \infty)$ and $\{f_\gamma\}_\gamma$, $\gamma > 0$, denote a net of functions in $L^p(\mathbb{R}^n)$ with $||f - f_\gamma||_p < \gamma$. We wish to determine a sharp scaling of ε and γ which guarantees that as $\gamma \to 0$

$$\phi(\varepsilon A)f = \frac{1}{2\pi i} \int_\Gamma \phi(z)\ (z - \varepsilon A)^{-1} f dz, \quad \text{contour } \Gamma,$$

converges to f in $L^p(\mathbb{R}^n)$, $1 \le p < \infty$ and pointwise on the Lebesgue set of f. Our main result provides a large class of L^p regularizing operators of the form $\phi(\varepsilon A)$ and a sharp condition on the associated scaling functions.

IV. New Project

7. with M. Kon and A. Ramm.

We are developing an abstract approach to the analysis of behavior of analytic functions, for example, resolvents and semi-groups, of perturbed elliptic operators. Specifically, we will consider the effects of perturbations on strong pointwise behavior of such functions in certain natural parameters. The prototypical example of a situation in which this is relevant is that of pointwise behavior of eigenfunction expansions.

Our motivation stems from the fact that it is easier to study expansions in eigenfunctions of unperturbed operators, for example, the Laplacian = $-\Delta$

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on a manifold, then those in terms of perturbed ones, say $-\Delta + q(x)$. Results which have been of interest in this context are equisummability theorems relating perturbed to unperturbed expansions pointwise. (See [S], [H]). We hope to extend this study to that of eigenfunction expansions under the action of multipliers.

Finally, for completeness we list the two co-authored papers published during this grant, but written under Raphael's former ARO grant.

V. Articles Published for ARO Grant #DAAG29-81-G0001.

8. with M. Kon: "New Multiplier Methods for Summing Classical Eigenfunction Expansions", Journal of Differential Equations, 50, no. 3 (1983), pp. 391-406. Status: ARO has twenty-five reprints.

9. with H. Diamond and B. Kuttner: "Inclusion Theorems for the Absolute Summability of Divergent Integrals", Canadian Mathematical Bulletin, 26, no. 4, 1983, pp. 389-398. Status: ARO has twenty-five reprints.

Raphael is extremely grateful to ARO for affording these opportunities to do research. During the span of this grant, she has enjoyed her most mathematically productive and creative period.

Abridged Bibliography

[APS] A. Andreev, V.A. Popov and B. Sendov, Jackson-Type Theorems for Best One-Sided Approximations by Trigonometrical Polynomials and Splines, Math. Notes of Acad. of Sci. of USSR (Mat Zametki), 26, (1979), pp. 889-896 [English].

[CSSS] C.K. Chui, O. Shisha and P.W. Smith, Best Local Approximation, J. Approx. Theory 15 (1975), pp. 371-381.

[DKR] H. Diamond, M. Kon, and L. Raphael, Stable Summation Methods for a Class of Singular Sturm-Liouville Expansions, PAMS 81 (1981) pp. 279-286.

[GK] D. Gurarie and M. Kon, Radial Bounds for Perturbations of Elliptic Operators. J. Functional Analysis (to appear).

[H] A. Haar, Zur theorie der orthogonalen Funktionensysteme, Math. Annalen 69 (1910), pp. 331.

[S] M.H. Stone, A Comparison of the Series of Fourier and Birkhoff, Trans. Amer. Math. Soc. 28 (1926) p. 695.

[W] J.L. Walsh, Pade Approximants as Limits of Rational Functions of Best Approximation, Real Domain, J. Approx. Theory II (1974), pp. 225-230.

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ACTIVITIES

1. Presented ARO sponsored research.
   a) International Conference on Differential Equations, Birmingham, Al. March, 1983.
   b) International Congress of Mathematicians, Warsaw Poland, Special Session on Partial Differential Equations, August 1983.
   c) Howard University, Department of Mechanical Engineering, October 1983.
   d) AMS meeting, San Luis Obispo, Ca., Special Session on Partial Differential Equations, November 1983.
   e) Bryn Mawr College, Pa., Fourier Series Seminar, December 1983.
   f) Howard University, Department of Mathematics, Spectral Theory Seminar, March and April 1984.
   g) Argonne National Laboratory, Spectral Theory of Sturm-Liouville Differential Operators, May 1984.
2. Meetings (In Addition to Above)
   a) Annual Meeting of the American Mathematical Society, Denver, Co. January 1983.
   b) International Conference on Approximation Theory, College Station, Tx., January 1983.
   c) ARO Conference, Washington, D.C. May 1983.
   d) ARO Workshop on Exterior Domain Problems in Mechanics, Howard University, June 1983.
   e) Annual Meeting of the American Mathematical Society, Liouville, Ky., January 1984.
   f) ARO Conference, RPI, Troy, N.Y., May 1984.

3. Work Sessions at Howard University.

   a) Harvey Diamond March 1983, September 1983, December, 1983

   b) Charles Chui and Harvey Diamond May 1983.

   c) Mark Kon, March 1983.

   d) Boris Schechtman, May 1983.

4. Work sessions at MIT

   a) Mark Kon July 1983, December 1983.

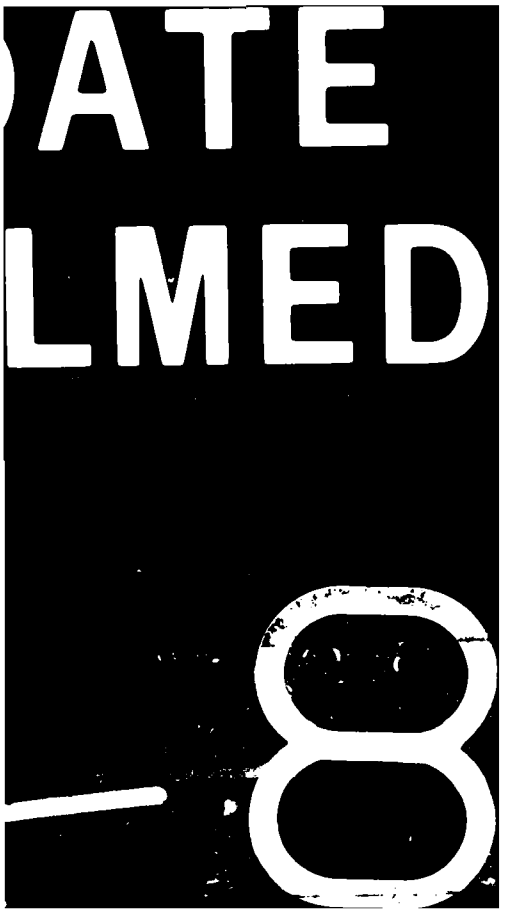
5. National Committee Appointments

   a) AMS representations to CBMS (Conference Board of Mathematical Sciences) Congressional Science Fellowship Panel

   b) Sloan Foundation Faculty Resource Consultant for new Liberal Arts Program

   c) AAAS Consultant, Indian colleges in Southwest, USA.

   d) *Consortuum on mathematics and its applications Project* (COMAP) National Council.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